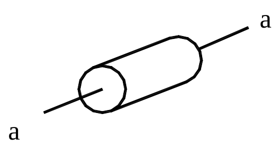
Inertia Calculations

Cylindrical object (solid) of diameter D (radius R=D/2) and mass, M



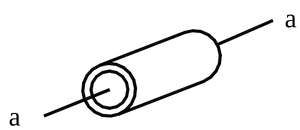
$$J_{aa} = \frac{1}{2}MR^2 = \frac{1}{8}MD^2$$
$$= \frac{1}{2}\pi\rho l R^4 = \frac{1}{32}\pi\rho l D^4$$

J_{aa} =mass moment of inertia about axis aa (polar moment of inertia)

ρ=density of the material

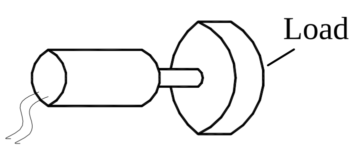
l=length of cylinder

Cylindrical object (hollow) with inner diameter D_i (radius R_i=D_i/2), outer diameter D_o (radius R_o=D_o/2), and mass, M



$$J_{aa} = \frac{1}{2}M(R_o^2 + R_i^2) = \frac{1}{8}M(D_o^2 + D_i^2)$$
$$= \frac{1}{2}\pi\rho l(R_o^4 - R_i^4) = \frac{1}{32}\pi\rho l(D_o^4 - D_i^4)$$

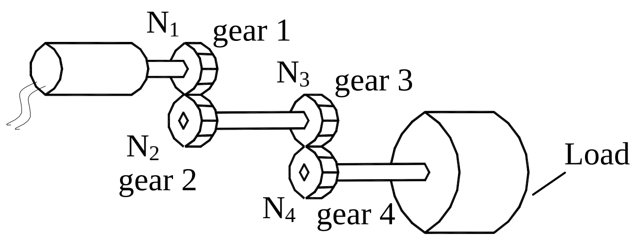
Direct drive load



J_{tot} = $J_{motor\ armature}$ + J_{load}

*Note: Shafts do have inertia, but their contribution to J_{tot} is often negligible. Why?

Gear driven load

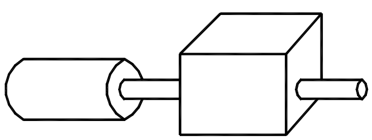


$J_{tot} = J_{motor\ armature} + J_{gear\ 1} + (N_1/N_2)^2\ [J_{gear\ 2} + J_{gear\ 3} + (N_3/N_4)^2\ \{J_{gear\ 4} + J_{load}\}]$

N_i is the number of gear teeth on gear i. N_i/N_j is the gear ratio between gears i and j.

(Note that the polar moment of inertia terms in the equation above refer to their central principal values about their axes of rotation)

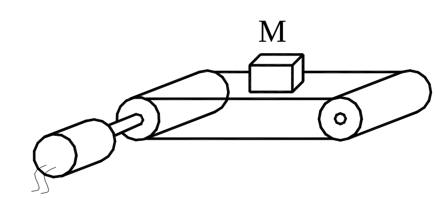
Leadscrew driven load



$$J_{tot} = J_{motor\ armature} + J_{leadscrew} + \frac{M}{(2\pi p)^2}\frac{1}{e}$$

p = leadscrew pitch (threads/length)
e = efficiency of leadscrew
M = mass of load
ρ = density of leadscrew material

Tangentially driven load



$J_{tot} = J_{motor} + J_{pulley1} + J_{pulley2} + MR^2 + M_{belt}R^2$

where $J_{pulley\ i}$ is the polar moment of inertia for pulley i about its rotational axis, M_{belt} is the mass of the belt, and R is the radius of both pulleys.