THE FORM OF THE STRESS-STRAIN CURVE OF CONCRETE INTERPRETED WITH A DIPHASE CONCEPT OF MATERIAL BEHAVIOUR*

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SUMMARY

Full stress-strain curves are given for concretes containing a range of dense and lightweight aggregates. An explanation of the difference in form of these relations is given in terms of induced stresses and it is shown that the stress-strain curves are members of a family of mathematical relations of simple form.

The reason for the difference between the origin normally adopted for stress-strain curves and the origin for the mathematical family is given. It is claimed that our basic model of the nature of a material, implicitly a single phase undifferentiated substance, is conceptually incorrect and that it needs to be replaced by a diphase model in which a quasi-solid phase in a high state of compressive strain is in equilibrium with a quasi-fluid in a high state of tensile strain. Adoption of this model leads naturally to a shift of stress and strain origin identical with that suggested by the mathematical form of the stress-strain curve. Some of the implications of this solid-fluid model are discussed.

1. INTRODUCTION

The structural properties of lightweight aggregate concretes have been extensively studied at the Building Research Station over the last decade. These studied have shown that lightweight aggregate concretes should not be regarded as materials essentially different from concretes made with dense aggregates, since they differ only quantitatively and not qualitatively. Consequently, lightweight aggregate concretes are now studied in the wider context of concrete as a material. The work described in this paper was undertaken to provide basic information on the full stress-strain curves of lightweight and dense concretes in compression. This information has a direct bearing on both the philosophy of design, i.e. whether an elastic or plastic method of design should be adopted, and on the relative toughness and resilience of these materials. Also, in so far as it can cast light on the failure mechanisms of concrete, it offers the prospect of increasing understanding of behaviour under fatigue and sustained load.

The work showed that the stress-strain characteristics of the concretes examined differed widely in form. At one extreme, for concrete with dense gravel aggregate the behaviour was plastic and at the other extreme for concrete containing the lightest of the lightweight aggregates, the concrete was brittle. The research also showed that the stress-strain curves up to failure were all members of a single family of mathematical relationships with the origin at the point of failure. Existing concepts of material behaviour fail to provide an explanation for this because they implicitly treat a material as a single phase unstressed and unstrained substance and ignore strain history and resulting internal stress states. It is suggested that replacing the existing conceptual model by a two-phase model in which all materials are regarded as two phases in mutually opposed states of strain induced during the formation of the material will result in a clearer understanding of material behaviour.

2. EXPERIMENTAL REMARKS

To ensure that the stress-strain curves related to materials as typical as possible of concretes used in practice, the size of compression specimen was chosen to be 300 mm × 150 mm square. A 1000-ton Amsler testing machine was chosen to load the specimens in compression. The stiffness of the basic machine is about 1500 ton/in, which was insufficient to follow steeply descending stress-strain curves. Various methods of stiffening the testing machine were considered but those used by previous investigators (Brock, 1962; Ramley, 1947) were not practicable in this case because of the very high loads involved. The eventual solution adopted was to test the specimen between two large concrete blocks 600 mm × 900 mm in cross-sectional area. Sandwich strips of steel-rubber-steel, 600 mm × 50 mm × 13 mm, were interposed between the tops of the concrete blocks and the testing platen. The stiffness of this layer could be varied over a large range by increasing or decreasing the gaps between the strips. It can be readily seen that for zero gap the rubber is in triaxial compression owing to the high shear generated and consequently its stiffness is increased considerably. The blocks and specimen were loaded using the 2400 mm long × 600 mm wide × 1200 mm deep wall platens—stiff plate girders—which are standard accessories for the Amsler testing machine (Figure 1).

The load measured by the ram of the testing machine indicated the total load on columns and specimen. In order to obtain the load imposed on

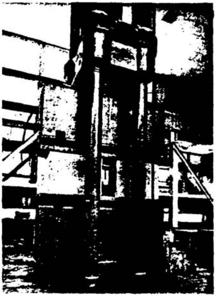

Figure 1. Experimental rig for determining full stress-strain curve of concrete

the specimen only, a load cell was placed in series with the specimen. Since this load cell had to be as stiff as possible to gain the full advantage of stiffening the machine, it was made from a 250 mm × 153 mm square block of steel turned down to 140 mm diameter over its central section. The load on this stiff cell was measured using 16 resistance strain gauges stuck to the central section of the cell. A wedge system and dry ball seating were also included in series with the specimen to take up slack and non-planarity before the commencement of the test.

The longitudinal deformations were measured by means of dial gauges measuring to 0·0025 mm and mounted on opposite faces of the specimens. The deformations measured were the overall deformations between the top and bottom platens. On some specimens internal longitudinal deformations were measured using 60 mm moulded wire strain gauges embedded in the centres of the specimens. Specimens were tested by loading at a constant rate of overall longitudinal deformation. The rate of loading was controlled by manually operating the valves on the load recording consul.

All concretes used in the tests were designed to have the same order of cube strength, which was about 30N/mm^2. Gravel was chosen as a representative dense aggregate and a range of lightweight aggregates (Aglite, Lytag, foamed slag and Leca) was also used.

3. FULL STRESS-STRAIN CURVES OF CONCRETES

Typical full stress-strain curves were obtained for a range of concretes (Figure 2). To avoid confusion, the stress-strain curves for Aglite and foamed slag concretes, which were similar to those for Lytag, have not been included on the graph. The stress-strain curves for gravel and Leca therefore represent the boundary cases of the range of concretes considered. The curves are presented on a comparative basis because this paper is concerned with the form of the stress-strain behaviour of concrete. The actual maximum stresses are all of the same order, about 24 N/mm^2 and the actual strains at maximum stress range from about 1500×10^{-6} for gravel concrete to 3000×10^{-6} for Leca concrete.

It can be seen from Figure 2 that at one boundary gravel concrete has a markedly curved stress-strain curve up to maximum stress and a shallow

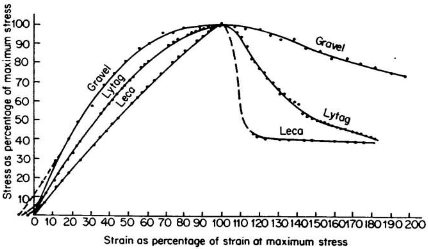

Figure 2. Comparative relations between stress and strain for 6 in square by 12 in long prisms of concrete

slope beyond, and at the other extreme Leca has an almost straight stress-strain curve up to failure and a steeply descending portion beyond failure. The former is characteristic of a plastic type of behaviour and the latter is characteristic of brittle behaviour.

4. FORM OF THE STRESS-STRAIN CURVE

It was found that, allowing for an initial bedding down region, the stress-strain curves were all power laws taking the point of maximum stress as the origin. The agreement between the mathematical form and the experimental points can be judged from Figure 3, where the full line is the best-fit power curve and the spots are the experimental results. The value of investigating concretes of widely differing properties can be clearly seen from Figure 3. The results for Leca and Lytag show quite clearly the existence of a lead-in part of the curve which can be regarded as a bedding down region similar to the lead-in that occurs in the CBR test (Road Research Laboratory, 1962), for example, and suggests that the lead-in portion for the gravel curve is a similar manifestation.

It seemed possible that the lead-in portion of the stress-strain curves was the result of slight uneveness in the ends of the specimen resulting in gaps which are closed up as the specimen is loaded. Internal cracks may also be present owing to differential shrinkage of the aggregate and matrix phases of the specimen (Glucklich, 1965; Hsul and coworkers, 1963). This reasoning suggested that stress-strain curves with no lead-in portion might be given by internal strain gauges. Accordingly a series of tests was carried out on specimens having a moulded wire strain gauge cast in the middle of the specimen. Typical results are given in Figures 4, 5 and 6 for a range of aggregates. It can be seen that there is good agreement between the power curves and the experimental points and that there is no significant lead-in portion.

5. THE EFFECT OF AGGREGATE-MATRIX INTERACTION ON THE FORM OF THE STRESS-STRAIN CURVE

The concrete can be regarded as a two-component system, namely a discontinuous aggregate component in a continuous matrix component. In general the aggregate component of the system will be stiffer than the matrix component, and it is evident that when such a system is subjected to a state of uniaxial stress this difference between the stiffness of the two components will set up an induced system of internal stress. These induced stresses will cause additional compressive stresses in the matrix at polar zones of the aggregate and tensile stresses in the matrix at the equitorial zones. As the external compressive stress is

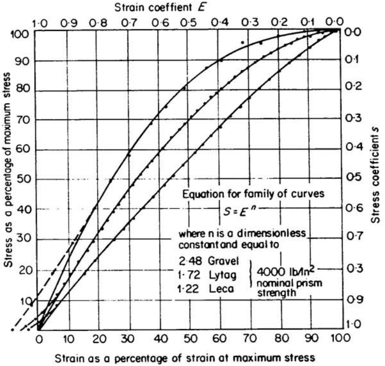

Figure 3. Comparative relations between stress and overall strain for 6 in square by 12 in long prisms of concrete

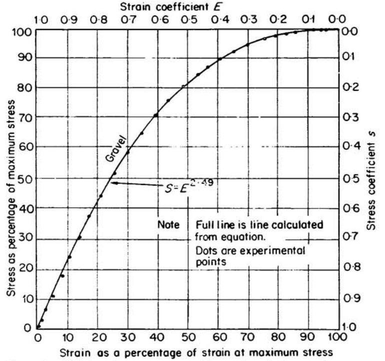

Figure 4. Relations between stress and internal strain for gravel concrete

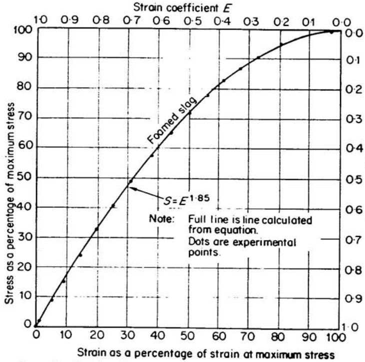

Figure 5. Relation between stress and internal strain for foamed slag concrete

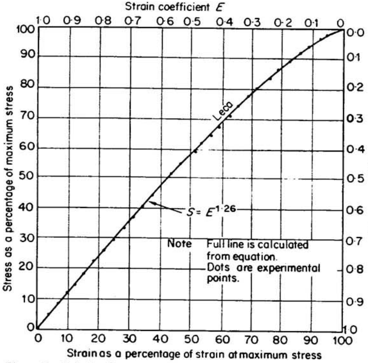

Figure 6. Relation between stress and internal strain for Leca concrete

increased, the equitorial tensile stresses will increase correspondingly and eventually tensile cracking parallel to the direction of external compression will take place. Progressive generation of such tensile cracks has been demonstrated by Jones (1952). The plastic behaviour resulting from this progressive vertical cracking gives rise to the observed curvature of the stress-strain curve. This explanation indicates that the rate of development of internal cracks will be dependent on the relative difference between the stiffness of the aggregate and that of the matrix since this will govern the magnitude of the induced stresses relative to the magnitude of the ambient stress.

It follows that as the aggregate stiffness approaches that of the matrix the curvature of the stress-strain curve will decrease; this agrees with the results obtained. It also follows that for a given aggregate, increasing the matrix strength and thereby reducing the stiffness differential between the matrix and the aggregate should decrease the curvature of the stress-strain curve. Examination of the stress-strain curves up to failure for a range of different concretes (Neville, 1963) containing the same aggregate shows that this is indeed so; the stronger the concrete, the more linear the stress-strain curve becomes. Therefore there is a stress-strain similitude between lightweight concretesand dense concretes of a higher strength. Since the lightest lightweight concrete tested failed in a very brittle fashion beyond the point of maximum stress it seems probable that high strength dense concretes will behave in a similar fashion.

In recent years there has been a move towards regarding concrete as a quasi-plastic material capable of sustaining high proportions of the maximum load at strains well beyond the strain at maximum stress. The present research suggests that this is only true for concretes having a low matrix-aggregate strength ratio and that a distinction needs to be made in design between concretes which are closer to the plastic assumption and those which are closer to the elastic assumption.

6. INTERPRETATION OF STRESS-STRAIN BEHAVIOUR IN TERMS OF A TWO-PHASE CONCEPTUAL MODEL

In the study of materials it is normally assumed that a material is a single phase undifferentiated substance; in other words that it is infinitely divisible rather than atomic. This assumption is never made explicitly and consequently its existence is rarely recognized. However, it is a very real assumption and it leads to a specific view of material behaviour. It is suggested that this assumption can be usefully replaced by the following assumption which gives a different frame of reference for understanding material behaviour.

A material may be regarded as a skeletal quasi-'solid' phase in a continuous quasi-'fluid' matrix, with the fluid phase in a high state of tensile strain relative to its free condition and the 'solid' in a state of balancing compressive strain.

Thus the 'solid' component which governs the shape of the material is in a state of triaxial stress resulting from the tension in the fluid phase. It is evident that the factor governing the strain state of the 'solid' phase is the potential drop between the external and internal environment and that if the internal 'fluid' were removed, an equal external compression of the 'solid' component would bring the solid to the same state of strain.

In such a two-phase system the strength, rigidity and other mechanical properties will depend on the value of internal stress. If this stress is reduced to zero the material will fall apart into its separate components. Failure is therefore seen as the point where the strain history of the material has been retraced to its origin and the point of failure becomes the natural origin from which to measure stress and strain.

This two-phase model can be extended indefinitely in a hierarchical sense. The quasi-'fluid' and quasi-'solid' components of the first order can themselves be divided into quasi-'fluid' and quasi-'solid' components of the second order, and so on. Clearly, such a process is impossible with the undifferentiated homogeneous model normally assumed for a material. With one class of engineering materials, soils, i.e. sands, silts and clays, the validity of the two-phase model can be demonstrated clearly since the tension in the fluid phase, in this case water, can be measured directly (Croney, 1952; Schofield, 1935) and shown to attain values of many thousands of pounds per square inch. Also, soils are a particularly simple case in that the strengths, or more generally the stress-strain properties, are determined basically by a single hierarchical system, i.e. the particle-water system. In the case of soils, therefore, the conceptual quasi-'fluid' corresponds to a readily identifiable chemical fluid (water) and the conceptual quasi-'solid' corresponds to a readily identifiable real solid (the soil particle structure). In general, however, the stress-strain properties of a material will not be dependent on a single hierarchical system but will be the result of the combined influence of several systems at different levels.

It is postulated that the behaviour of each hierarchical system is essentially similar, and because of this the multitude of systems can be replaced by a single equivalent system in which the total effect of the different orders of 'solids' involved are represented by a single equivalent 'solid' and the total effect of the different orders of 'fluids' involved are represented by a single equivalent 'fluid.'

6.1. Effect of an External Stress on a Two-phase System

Consider the effect of subjecting a two-phase system to an external stress, for example an external uniaxial compression.

The external load will be partitioned between the quasi-'solid' and the quasi-'fluid' components of the material, resulting in an increase in the compression strain of the solid structure and a decrease in the tensile strain of the fluid. The consequent reduction in the fluid hydrostatic pressure will result in a reduction in the pressure difference across the lateral faces of the specimen and a resulting lateral expansion. The change in the fluid tension is in effect a shift of the internal frame of reference for stress in relation to the external frame of reference for stress. Viewed from the internal frame of reference the effect of an external uniaxial compression is to reduce the lateral compressive stress, which is equivalent to imposing a lateral tensile stress. Viewed from the internal frame of reference, therefore, lateral strains are a logical necessity, a direct outcome of stress changes. But viewed from the external frame of reference, there is no lateral stress and lateral strains are merely a nuisance, an unwanted effect which complicates calculations and is ignored or neglected wherever possible.

As the external uniaxial compression is increased the later pressure difference will continue to decrease until it is zero. At this point there will be no lateral pressure difference holding the elements of the specimen together and it will divide vertically into a number of pieces.

Viewed from the internal frame of reference the lateral tensile stress has increased (equivalently, but more correctly, the lateral compressive stress has decreased) until the material has failed laterally in tension. Viewed from the external frame of reference, there is no lateral stress and so one of the most universal of all failure phenomena, the failure in compression by vertical splitting, has to be either ignored or explained away with vague statements about secondary stresses.

6.2. Experimental Support for Concept of Two-phase Model

It has been shown that the stress-strain curves for a range of concretes can be represented by simple power laws which involve a single constant for each curve and take the point of maximum stress as the origin. However, the origin normally used for viewing stress-strain phenomena does not lead to any simple mathematical expressions.

Adoption of the two-phase hierarchical model suggests, firstly, that the origin of the stress-strain curves of concrete is the point of failure since this corresponds to the internal frame of reference. This is faithfully reflected by the mathematics of the family of relationships for which the point of maximum stress is the mathematical origin. Secondly, the model suggests that the relation will be of a simple mathematical form, since though the overall stress-strain characteristic is the product of many hierarchical systems their net effect is equivalent to the effect of a single system. This is borne out by the fact that within the limits of experimental error the form of the stress-strain curve is expressed by a single constant. Thirdly, it suggests that since the hierarchical subsystems are all essentially similar, any compound of them into a single equivalent system must give rise to essentially the same form of behaviour. This is so since all the stress-strain curves belong to the same mathematical family, a family of power curves.

Adoption of the two-phase model also suggests that failure in compression will occur by vertical division. Experimentally this can easily be demonstrated by using Hilsdorf platens (Hilsdorf, 1965) which do not induce lateral compressive stresses at the ends of the specimens. Hilsdorf brushes with 3 mm square steel hairs (Figure 7) have been used to crush concrete specimens and in all cases failure occurs by vertical division. Shear failure is evidently a secondary phenomenon

Figure 7. Hilsdorf brush platens

caused by externally or internally induced lateral stresses which distort the uniform stress field. The primary nature of tensile failure is illustrated by the failure of gravel concrete, for example; internal tensile cracks parallel to the direction of compressive stress can be demonstrated to exist long before any shear plane develops (Jones, 1952).

There are numerous other examples of internal stress systems both natural and artificial where the stress-strain properties of the composite are dependent upon the mutually opposing internal stresses. Sometimes imposed stress systems provide the only source of strength as in prestressed or inflatable structures. In other cases the stress system is imposed upon an existing system as in toughened glass. The existence of some internal stress systems can be demonstrated visually, e.g. the frozen-stress patterns employed in photoelastic stress analysis. It is relevant to note that these stress patterns are accounted for by a 'diphase molecular structure for Bakelite' (Frocht, 1941) and that this allows the frozen stresses to be viewed as real stresses and not fictitious stresses.

Much more qualitative evidence could be advanced to demonstrate the correctness of the two-phase hierarchical concept of material behaviour and the necessity for considering the total stress (i.e. the sum of the internal and external stress) on the particular material hierarchy under consideration and not just the external stress as at present. However, these qualitative arguments depend considerably on personal inclinations and are therefore of limited value. Similarly, the experimental data relating to material behaviour usually contain relatively large experimental errors. Different mathematical forms will therefore provide good fits for the data. A classical example of this situation occurred with Dulong and Petit's results for the relation between the quantity and quality of full radiation and temperature. Dulong and Petit proposed an exponential form and this fitted the results quite well, but Stefan, on examining the results, found they accorded even better with a fourth power law. Subsequently this fourth power law was derived theoretically by Boltzmann. Evidently a few key quantitative examples of the power of the model to arrive at a new and simple understanding of material behaviour will be of more value than a much larger volume of qualitative argument. One such example is given below.

6.3. The Relation Between Pressure and Volume of Water at Constant Temperature

In a 'solid-fluid' hierarchical system both the nth order 'solid' and 'fluid' components can be considered as materials comprised of 'solid' and 'fluid' components of the $(n + 1)$th order. For example, in a clay-water system the clay particle structure and the water are the quasi-'solid' and quasi-'fluid' of the first order. Dropping a hierarchy and considering next the water, the water molecules and the van der Waals space are the solid and fluid components of the second order. If this concept is valid then the van der Waals space can be treated as a fluid in a high state of tension and the water molecules as a skeletal structure in a balancing state of compression. An external stress on the water will be added to the internal stress to give a total stress and this value of total stress will act upon the skeletal molecular structure.

Since the model postulates that the behaviour of each hierarchy is essentially similar, it follows that all physical relations measured from true origins must have the same mathematical form whether they are the product of a single hierarchy or a combination of hierarchies, i.e. they must all be power laws. Therefore let us assume that the relation between total stress, P, and total volume, V, for water is of the form

$$P = -kV^n$$

where $P = p + i$, p = external stress and i = internal stress applied by van der Waals fluid;

whence

$$(p + i) = -kV^n$$

$$\log(p + i) = -(n \log V + \log k)$$

$$\frac{dp}{(p + i)} = -n\frac{dV}{V}$$

Therefore by plotting $V(dp/dV)$ against p it is possible to determine both n, the power of the relation, and the negative intercept i, the internal stress. This was done for water using Bridgman's data taken from the International Critical Tables (Bridgman, 1928) which gives the relation between pressure and volume for water up to pressures of 12,000 atmospheres. It was found that $n = 6$ and that the internal pressure was of the order of thousands of atmospheres. The relation for 60°C between the total pressure (taking the internal pressure as 3750 atmospheres) and the experimental points is so good (Figure 8) that it is necessary to show the deviations between the calculated and the actual values by a subsidiary graph in which the deviations are magnified by a factor of 10. Also, the fact that the power is an integral value suggests that the underlying mechanisms are relatively simple and the analysis of the relation into simpler parts should not be too difficult.

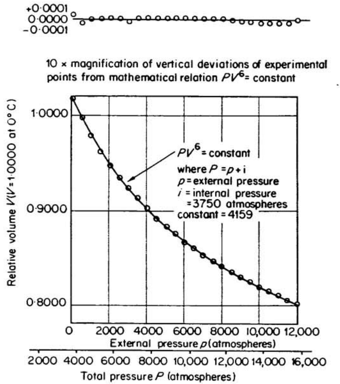

Figure 8. **Relation between pressure and volume for water at 60°C (Bridgman's experimental data)**

6.4. Non-existence of Tensile Forces

It has been shown that the adoption of a two-phase concept of a material leads to a simpler interpretation of material behaviour. So far the model has assumed a solid phase in compression and a fluid phase in tension. However, since the fluid phase is itself a material, this too must have a tension-compression balance. These tensions can be looked at in a different way. By taking the zero as the value of the tensile stress in the fluid phase we are effectively treating the fluid phase as though it is initially in compression, but when incorporated with the solid phase this compression is reduced. Implicitly, then, we are treating the material as though it is surrounded by an external fluid which is pressing on it in a way similar to the atmosphere, but with in general a very much higher pressure. If this viewpoint is adopted, there is no such thing as a tensile force; all forces are compressive. Exactly the same situation arises with temperature. We do not have positive and negative temperatures but only positive ones and this approach leads to simple relations between phenomena and temperature, e.g. Stefan's law. The situation is precisely the same except that it is in a different solid/fuel hierarchy of phenomena. A tensile stress, therefore, is merely a reduction in the ambient compressive stress and the concept of action at a distance is no longer required. It is interesting to examine a few phenomena to see if these concepts can lead to any new understanding.

6.5. Compressive and Tensile Strengths of Brittle Materials

Consider a material to be composed of solid particles which are under compression by a surrounding external hydraulic fluid. Under an additional compressive stress, applied by a steel platen, say, the internal fluid pressure will be raised which will lead to a reduction in the compressive stress laterally. Vertical cracks will form and these cracks will be open to the external hydraulic pressure which is the lower of the two external pressures. In a tensile test, however, the platen pressure will be the lower pressure and the cracks will be at the higher of the two external pressures which is the hydraulic ambient pressure. Essentially, therefore, the difference between the compressive strength and tensile strength of a brittle material is the difference in the relative crack pressure. Where the external hydraulic pressure cannot get into the specimen the tensile and compressive strengths will be virtually the same.

It ought to be possible therefore to measure the tensile strength of a material by applying a biaxial compression with a hydraulic fluid. The tensile strength of materials has in fact been unintentionally measured in this way by Bridgman (1949) who stated: 'When the pressure rises to a value approximately equal to the breaking stress in pure tension the cylinder parts as though it had been pulled apart by a tensile pull applied to the projecting ends. If the rod is of a brittle material like glass or a glass-hard tool steel the rupture takes place on a perfectly clean plane perpendicular to the axis, but if the rod is of a material that can yield before rupture like soft steel there is a considerable contraction at the area of the break which looks like the break in an ordinary tensile test.'

Thus the fracture behaviour of materials is consistent with their being under a high ambient pressure applied by a quasi-fluid. This may be called a beta-atmosphere to distinguish it from the normal atmosphere we are familiar with, which exerts $0{\cdot}1\,N/m^2$ and may be designated the alpha-atmosphere.

6.6. Propagation of Sound and Light

Qualitatively, light and sound exhibit precisely the same behaviour. For example, a sound ray will suffer refraction when passing from a cold to a hot layer of air. In the case of sound there is a perfectly comprehensible explanation for this wave behaviour based upon changes in carrier velocity. It is suggested that light also has a carrier, the beta-atmosphere particle, and that the pressure drop across a material boundary arises from a difference in R.M.S. velocity.

6.7. Osmotic Pressure

The fact that the behaviour of a dissolved substance is the same as the behaviour of a gas and follows the same gas laws with the same constant R strongly suggests that a gas is a dissolved substance, viz. an alpha-atmosphere dissolved in a beta-atmosphere and that 'atmospheric pressure' is the osmotic pressure of the alpha-atmosphere.

6.8. Physical Relations as Power Curves

It has been observed that physical relations measured with respect to their true origins are power curves. Essentially these power curves arise out of a conceptual error which is similar to the error about origins. It is assumed that the concept of length applying outside a material also applies inside a material. However, when a material changes relative to its environment the concepts of length inside and outside the material change relative to each other. The concept of length is grounded in an operational specification in precisely the same way as the concept of change (time). Put in another way, length is nothing more nor less than what a ruler measures; the fact that the ruler is shortening, lengthening, bending or suffering any other kind of distortion relative to some other ruler is irrelevant since the second ruler is suffering a symmetrical distortion relative to the first and it is meaningless to talk of which is being distorted without some absolute standard of reference. If the idea of an external absolute standard of length is abandoned and we take an internal standard (the length of the object, say) then the mathematical relations governing change become simpler. For example, considering the changes taking place during the compression of water, the internal unit of change becomes dl/l and recognizing that pressure is phenomenologically merely an external length change, the power relations of physics are seen to be of the form

$$M_a = -k N_b \qquad M_a = -kN_b$$

where M_a and N_b are changes with respect to internal standards and k is a constant.

It seems probable that the constant k arises from the fact that though the hierarchical system is open-ended each hierarchy is quantitized and the quantitization of relative length in different hierarchies and different materials is different. The question arises, 'Is there some fundamental concept which does not require a conversion factor?' It is suspected that the answer is 'yes' and this concept is the bit, the unit of information. This concept is numeric and involves no dimension other than number. The basic conservation equation of change between two hierarchies then becomes

$$Ia = -Ib \qquad I_a = -I_b$$

and involves no arbitrary constants.

The two hierarchies combined constitute a global hierarchy which contains more information than the sum of the information of the two hierarchies because it contains the information arising from the interaction between them. It is suspected that the principle governing the direction of change between the two inferior hierarchies is that the information content of the superior global hierarchy always increases.

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