

PEQUAIDE'S CYLINDER AND SPHERES

Preliminary May 12, 2007 by Tom Schum

FUNDAMENTALS

KINETIC ENERGY

The formula "given" for kinetic energy is $E = 1/2 m(v^2)$ (energy = velocity squared, multiplied by mass, multiplied by one-half). Units of energy are j (joules = newton-meters).

This "kinetic energy" claims to be the same as "work", which is defined $W = FD$ (work = force multiplied by distance). Units of energy are the same j (joules = newton-meters).

For example a 1 kg object falls for 1 second and the total distance moved is 4.9 meters. The acceleration due to gravity is 9.8 m/s/s (9.8 meters per second per second). This means the force of gravity is 9.8 n/kg (9.8 newtons per kilogram). If the fall time is 1 sec, speed at the end of the fall is 9.8 m/sec. The distance traversed during this fall is $D = 1/2 a(t^2) = 4.9$ meters. Work done is then 9.8 n/kg force multiplied by 4.9 meters distance = 48.02 joules.

Taking the speed of our object, 9.8 meters/sec, and the mass of 1 kg, the kinetic energy formula gives us an answer of 48.02 joules. Our two methods of calculation deliver the same result in this case.

MOMENTUM

The formula "given" for momentum is $M = mv$ (mass multiplied by velocity). Units are kg-m/sec, where mass is in kilograms, and velocity is in meters per second. In our example, we have a 1 kg object moving at 9.8 m/s, so the momentum is 9.8 kg-m/sec.

BASIC COLLISION THEORY

It is well-known that in collisions momentum is conserved. That is, the total momentum of the system in collision is the same before the collision as after the collision. For an example, consider a system consisting of two billiard balls on a pool table. If you accelerate one of them (by means of a pool cue) so that it hits the other one, the results of the collision will not change the system momentum.

An additional caveat is currently accepted, which is that the total kinetic energy in a system will be the same before and after a perfectly elastic collision. We now have various "qualities" of collision to define. An inelastic collision is one in which there can be no rebound, such as when a bullet hits a sand dune. Instead of rebounding, the bullet heats the sand as it is slowed by it. An elastic collision, on the other hand, involves a rebound. In a perfectly elastic collision there is no heat produced. Note that in both kinds of collision momentum is always conserved.

We are working with perfectly elastic collisions in this document, because it is simpler. There are no losses in which kinetic energy is turned into heat.

THE END OF THE RULE OF KINETIC ENERGY

Pequaide's cylinder and spheres experiment starts with a massive system in motion, and ends with two spheres in faster motion. Momentum is conserved, but kinetic energy is not, because the system momentum ends up concentrated in a small portion of the original mass of the system.

For example, consider the following.

Case 1: If a 1kg (kilogram) mass is moving at a velocity of 10 m/sec (meters per second) its momentum is mass multiplied by velocity = 10 kg-m/sec.

Case 2: If a 200 gram mass is moving at a velocity of 50 m/sec, its momentum is 10 kg-m/sec.

The momentum is the same in both cases, but kinetic energy in these two cases is far different. Kinetic energy of the 1kg mass in case 1 is 50 j. Kinetic energy of the 0.2 kg mass in case 2 is 250 j. So for a given amount of momentum, the smaller mass will store more energy.

Pequaide's cylinder and spheres experiment concentrates the system momentum into a small portion of the system mass. Therefore, the kinetic energy equation fails.

THE BASIC PROBLEM

How can the momentum of a massive system be concentrated into a small portion of the system mass without expenditure of energy?

ENGINEERING THE APPROPRIATE COLLISION

There is a precedent. A scientific "toy" exists which does something quite similar. It consists of three balls on a string. The bottom ball is largest, the top ball is smallest, and the middle ball is in between these sizes. The string is secured to the earth, and held vertical. The balls are dropped in a group down the string. When they hit the ground, the smallest ball bounces far higher than it could bounce by itself. The small ball stores some of the momentum of the other two, and bounces higher because of this. We need to understand why this happens, and see if we can engineer a one-dimensional momentum concentrator. Then we can move on to advanced rotary solutions such as those designed by Pequaide.

ONE-DIMENSIONAL COLLISIONS

A bola consists of two spheres on a string. We are talking about only one of many forms of the bola, the Avestrucera o nanducera (2 weights). See the following link for more information: <http://www.flight-toys.com/bolas.htm>

The spheres joined by the string (or tether) can be interacted with simultaneously. Just grab the string in the middle with one hand and start moving your hand. With the correct up-and-down hand motion the bolas can be accelerated so they move in semicircles and collide with one another at the top and bottom of their travel. Many bolas would break apart if operated in this way because the weights cannot stand these continuing impacts, so don't try this on the Pampas.

This same sort of collision can be seen in a Newton's Cradle. Pull back the spheres on each end and release them simultaneously. The spheres in the middle will not move, but the two moving spheres will bounce off as if they are hitting an immovable object. This bouncing will continue for quite a while. If you remove the middle spheres from the setup, this bouncing action will work just fine with two spheres.

So, we know if two spheres having the same mass and exactly opposite velocity collide with one another, they will bounce off each other. The momentum of one sphere ends up passing to the other, and vice versa. In order for this to happen, the momentum of each mass must reverse as a result of the collision. This requires an impulse force strong enough to impart twice the momentum currently held in the sphere, and short enough to do this in less than a millisecond.

The same thing happens if one sphere hits an immovable surface such as a very hard and very massive block of solid steel anchored to the earth. The sphere is "reflected" on impact.

CENTER OF MASS = VIRTUAL IMMOVABLE OBJECT

See figure 1 in the figures section after this text. In figure 1 the center of mass of the system is called "the virtual immovable object". The figure shows two spheres of identical mass and equal and opposite momentum colliding head-on at the center of mass. Since the spheres are of equal mass, the center of mass of the system is at the middle of the line drawn between the two. Soon we will move on to sets of colliding spheres with different masses, so keep this in mind.

Suppose the center of mass of the system in figure 1 (the virtual immovable object) is moving -10 m/sec, and each sphere is moving at 10 m/sec relative to that center of mass (same as before). Velocity of sphere A is 0 m/sec and velocity of sphere B is -20 m/sec. After the collision, velocity of sphere A is -20 m/sec and velocity of sphere B is 0 m/sec. This is what happens in a Newton's Cradle, except that pendulum action keeps bringing the spheres back into collisions, producing a cycle.

Now, suppose the center of mass of the system in figure 1 is moving -5 m/sec, and each sphere is moving 10 m/sec relative to that center of mass. Velocity of sphere A is 5 m/sec, and velocity of sphere B is -15 m/sec. After the collision, velocity of sphere A is -15 m/sec and velocity of sphere B is 5 m/sec.

Our little plan to take over the world is not working very well so far. Time to brush up on first principles (again).

COLLISIONS BETWEEN SPHERES OF DIFFERENT MASS

See figures 2 and 3.

Figure 2 is developed from the Wikipedia example cited. The numbers don't seem to make a lot of sense intuitively, until we realize that the system center of mass is moving at -2.25 m/sec. Then we can develop figure 3 by just adding 2.25 m/sec to all the velocities, before and after the collision. Now the system center of mass is stationary.

Suddenly, the dynamics of this collision become crystal clear. A reflection occurs at the center of mass, and it is just that simple. The waters were made quite murky by somebody throwing in an arbitrary system state of motion. Wiki is wonderful, however, because now we have an understanding of how system motion relates to the fixed frame.

Also, now we understand what happens when two spheres of different mass collide on a one-dimensional path. This explains what happens if you go to a Newton's Cradle and replace one of the spheres with a larger one. But we are drifting astray here. Back to the subject matter!

THE ENGINEERING GOAL

So, in a system moving relative to another system (the "fixed frame", where we sit watching all this), a collision is desired that will leave one sphere motionless relative to the fixed frame. Also, before this collision we want both spheres to be moving relative to the fixed frame. In the moving system the two spheres have a collision, and one of the spheres ends up motionless in the fixed frame. From our viewpoint in the fixed frame, the moving sphere has carried off all the momentum of the system since the motionless sphere has no momentum.

Taking another look at figure 3 we see that the velocity of the larger sphere is -3.75 m/s before the collision, and +3.75 m/s after the collision. This is a difference of 7.5 m/s. We want the larger sphere to be stopped after the collision, so it needs to be moving at -7.5 m/s before the collision. We have to add -3.75 m/s to all the velocities before and after the collision to preserve the dynamics. See figure 4.

So, in figure 4 we see a collision in which all the momentum and kinetic energy in the system is carried off by the smaller mass. Unfortunately however, no laws of physics are violated.

There is no specific requirement about the masses involved, or anything like that. This collision can be engineered to meet the requirements.

INVERSE COLLISIONS USING TETHERED SPHERES

Look again at figure 3. If all the velocities are reversed, and the two spheres are joined by a perfectly elastic tether that has no mass at all, an inverse collision will occur when the tether tightens. See Figure 5 for a depiction of this collision.

We can approximate the performance of this tether with a steel guitar string or other high-strength high-elasticity tether. So, this inverse collision idea can be verified experimentally.

To engineer this collision so that the larger mass is left stationary, we need to cause the system to be in motion at +3.75 m/s, exactly opposite to the velocity we used in figure 4.

When we try to use a rotary device, the geometry of the rotating frame of reference will produce forces for us which will lead to inverse collisions.

GEOMETRIC DYNAMICS OF A ROTARY DEVICE

Refer to figure 6. At the top left we see a rotating cylinder. Next, we look straight down on it as it rotates. All the mass in the cylinder is concentrated at the rim. As the cylinder rotates, if a particle is released from the outer surface of the cylinder, it will move on a tangential line. The next drawing in figure 6 is a simplified view of Pequaide's cylinder with spheres embedded. After that, we see how the spheres will move when released.

In figure 7, we look more closely at the geometry, throw in some arbitrary numbers for initial speed and size of the cylinder, and find out how this geometry gives rise to a surprisingly large difference in rotational speeds between the cylinder (continuing to rotate as before) and the system consisting of the spheres, now rotating far more slowly. This difference in rotational speeds arises in less than 1/10 second in this example, and is a difference of 290.5 RPM.

We also see tethers between each sphere and the center of system rotation. It is clear that the tether is somewhat longer than the distance moved by the sphere. This means a practical device of this type can be made in the real world.

The tethers pass thru slots in the cylinder (not shown). The slot dimensions are such that the tethers will establish a collision between the two systems before the two spheres have had a chance to have their inverse collision as the tethers tighten. This occurs when the tether hits the end of the slot. The tether hits the end of the slot very slightly before the two tethers tighten against each other (we want to prevent this). Figure 7 shows the tethers tight against each other, a situation that will be prevented in a useful device.

MOMENTUM IN THE ROTARY DEVICE

If we arbitrarily assign masses to the various parts of the device, we can calculate momenta and kinetic energies. Suppose the spheres, made of steel, have a mass of 0.1 kg each, and the cylinder, made of PVC pipe, has a mass of 0.4 kg.

We know the cylinder mass is concentrated at the rim, so we will use the tangential velocity which is 2 m/s, and the momentum of the cylinder is 0.8 kg-m/sec.

The spheres each move at 2 m/s after they are released, and until the tethers tighten. Just before the tethers tighten, the system will be in the configuration shown in figure 7, and the spheres will still be moving at 2 m/s. Their momenta are equal and opposite, 0.2 kg-m/sec each. At this point, 1/3 of the system momentum is in the spheres, and 2/3 is in the cylinder, which is the same as the momentum of the system before the spheres were released.

FORCES IN THE ROTARY DEVICE

Refer to figure 7. If the tether is tight against the end of the slot in the cylinder, an inverse collision between the cylinder and spheres will occur.

Figure 8 shows the new tether length and the new paths for the spheres at the first instant when the tether tightens against the end of the slot in the cylinder. This is a depiction of the instant of inverse collision between the cylinder and the spheres.

If we wish to follow the action from here on out, we will have to re-draw this figure for every millisecond, then find the differences, then calculate the forces involved.

A lot goes on here, after which we get to the following....

PEQUAIDE'S SOLUTION

Pequaide achieves this sort of momentum concentration in a rotary system, acting in the horizontal plane. The larger system is a cylinder with two identical spheres embedded in its wall. The smaller system is the two identical spheres alone. Pequaide has engineered the mass of the cylinder, the mass of the spheres, and the lengths of tethers for the spheres, so that when the spheres are released, they bring the rotation of the cylinder very quickly to a stop.

If the tethers were cut at the instant the cylinder rotation stops, the spheres would carry off all the momentum of the system. Together, the two spheres would carry a quantity of kinetic energy greater than the total system kinetic energy before the concentration event. Momentum would be conserved, however. If total kinetic energy before the concentration event were 50 j, and after the concentration it became 250 j, the free gain would be 200 j. Either energy appears from nowhere, or the kinetic energy formula is defective.

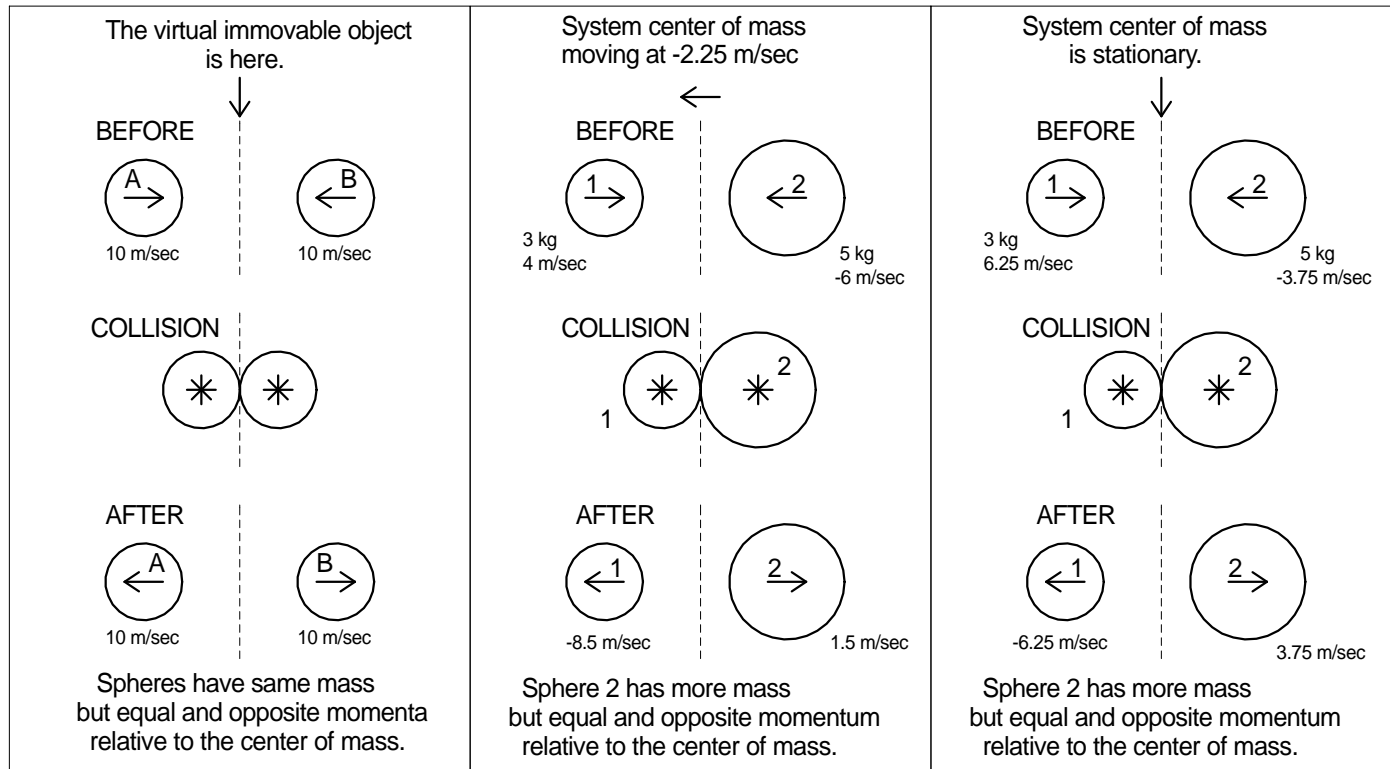


FIGURE 1

FIGURE 2

FIGURE 3

Wiki example:

(we calculate a few more results using the original Wiki data)

Before collision:

Ball 1: mass = 3 kg, $v = 4$ m/s
momentum = 12 kg-m/s
KE of ball 1 = 24 J
Ball 2: mass = 5 kg, $v = -6$ m/s
momentum = -30 kg-m/s
KE of ball 2 = 90 J

Adding all momenta = -18 kg-m/s
system mass = 8 kg
system velocity = -2.25 m/s
Total KE = 114 J

After collision:

Ball 1: mass = 3 kg, $v = -8.5$ m/s
momentum = -25.5 kg-m/s
KE of ball 1 = 108.375 J
Ball 2: mass = 5 kg, $v = 1.5$ m/s
momentum = 7.5 kg-m/sec
KE of ball 2 = 5.625 J

Adding all momenta = -18 kg-m/s
system mass = 8 kg
system velocity = -2.25 m/s
Total KE = 114 J

Wikipedia example (for reference) :

Ball 1: mass = 3 kg, $v = 4$ m/s
Ball 2: mass = 5 kg, $v = -6$ m/s
After collision:
Ball 1: $v = -8.5$ m/s
Ball 2: $v = 1.5$ m/s

link:

http://en.wikipedia.org/wiki/Elastic_collision#Equations_and_calculation_in_the_one-dimensional_case

modified Wiki example:

(we add 2.25 m/s to all initial and ending velocities)

Before collision:

Ball 1: mass = 3 kg, $v = 6.25$ m/s
momentum = 18.75 kg-m/s
KE of ball 1 = 58.59375 J
Ball 2: mass = 5 kg, $v = -3.75$ m/s
momentum = -18.75 kg-m/s
KE of ball 2 = 35.15625 J

Adding all momenta = 0 kg-m/s
system mass = 8 kg
system velocity = 0 m/s
Total KE = 93.75 J

After collision:

Ball 1: mass = 3 kg, $v = -6.25$ m/s
momentum = -18.75 kg-m/s
KE of ball 1 = 58.59375 J
Ball 2: mass = 5 kg, $v = 3.75$ m/s
momentum = 18.75 kg-m/sec
KE of ball 2 = 35.15625 J

Adding all momenta = 0 kg-m/s
system mass = 8 kg
system velocity = 0 m/s
Total KE = 93.75 J

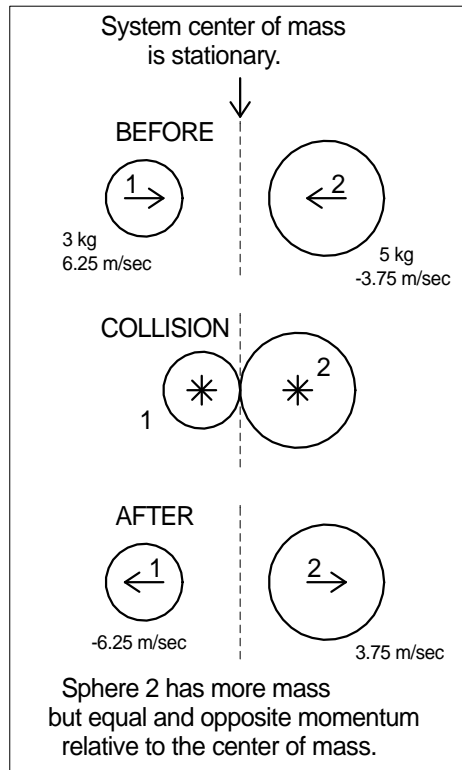


FIGURE 3
(for reference)

modified Wiki example:
(center of mass at rest)

Before collision:

Ball 1: mass = 3 kg, $v = 6.25$ m/s
momentum = 18.75 kg-m/s
KE of ball 1 = 58.59375 J
Ball 2: mass = 5 kg, $v = -3.75$ m/s
momentum = -18.75 kg-m/s
KE of ball 2 = 35.15625 J

Adding all momenta = 0 kg-m/s
system mass = 8 kg
system velocity = 0 m/s
Total KE = 93.75 J

After collision:

Ball 1: mass = 3 kg, $v = -6.25$ m/s
momentum = -18.75 kg-m/s
KE of ball 1 = 58.59375 J
Ball 2: mass = 5 kg, $v = 3.75$ m/s
momentum = 18.75 kg-m/sec
KE of ball 2 = 35.15625 J

Adding all momenta = 0 kg-m/s
system mass = 8 kg
system velocity = 0 m/s
Total KE = 93.75 J

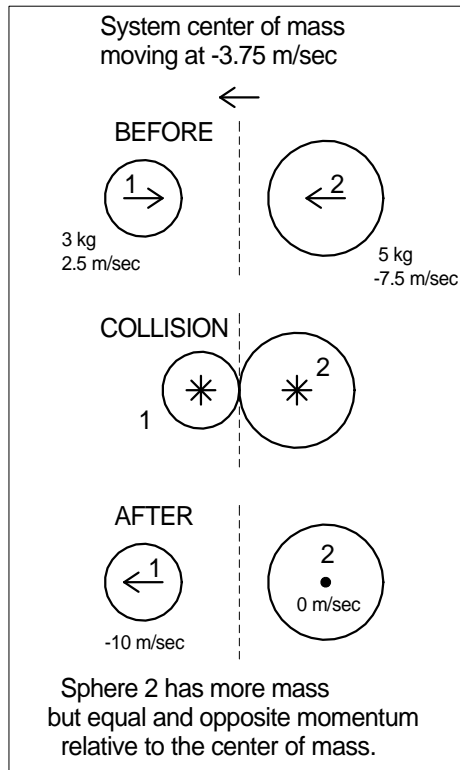


FIGURE 4

Our engineered collision:
(we add -3.75 m/s to all initial and ending velocities)

Before collision:

Ball 1: mass = 3 kg, $v = 2.5$ m/s
momentum = 7.5 kg-m/s
KE of ball 1 = 9.375 J
Ball 2: mass = 5 kg, $v = -7.5$ m/s
momentum = -37.5 kg-m/s
KE of ball 2 = 140.625 J

Adding all momenta = -30 kg-m/s
system mass = 8 kg
system velocity = -3.75 m/s
Total KE = 150 J

After collision:

Ball 1: mass = 3 kg, $v = -10$ m/s
momentum = -30 kg-m/s
KE of ball 1 = 150 J
Ball 2: mass = 5 kg, $v = 0$ m/s
momentum = 0 kg-m/sec
KE of ball 2 = 0 J

Adding all momenta = -30 kg-m/s
system mass = 8 kg
system velocity = -3.75 m/s
Total KE = 150 J

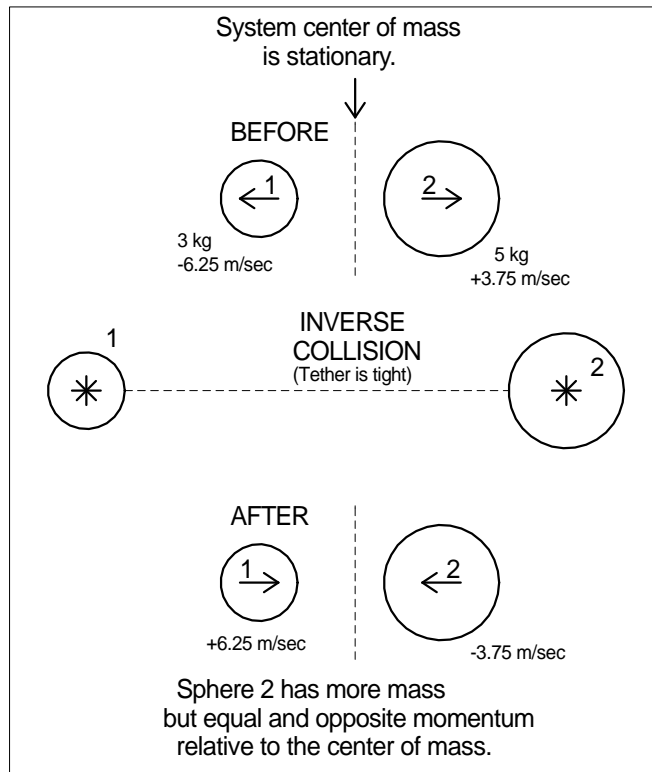


FIGURE 5

Tethered inverse collision:
(center of mass at rest)

Before collision:

Ball 1: mass = 3 kg, $v = -6.25$ m/s

momentum = -18.75 kg-m/s

KE of ball 1 = 58.59375 J

Ball 2: mass = 5 kg, $v = +3.75$ m/s

momentum = $+18.75$ kg-m/s

KE of ball 2 = 35.15625 J

Adding all momenta = 0 kg-m/s

system mass = 8 kg

system velocity = 0 m/s

Total KE = 93.75 J

After collision:

Ball 1: mass = 3 kg, $v = +6.25$ m/s

momentum = $+18.75$ kg-m/s

KE of ball 1 = 58.59375 J

Ball 2: mass = 5 kg, $v = -3.75$ m/s

momentum = -18.75 kg-m/sec

KE of ball 2 = 35.15625 J

Adding all momenta = 0 kg-m/s

system mass = 8 kg

system velocity = 0 m/s

Total KE = 93.75 J

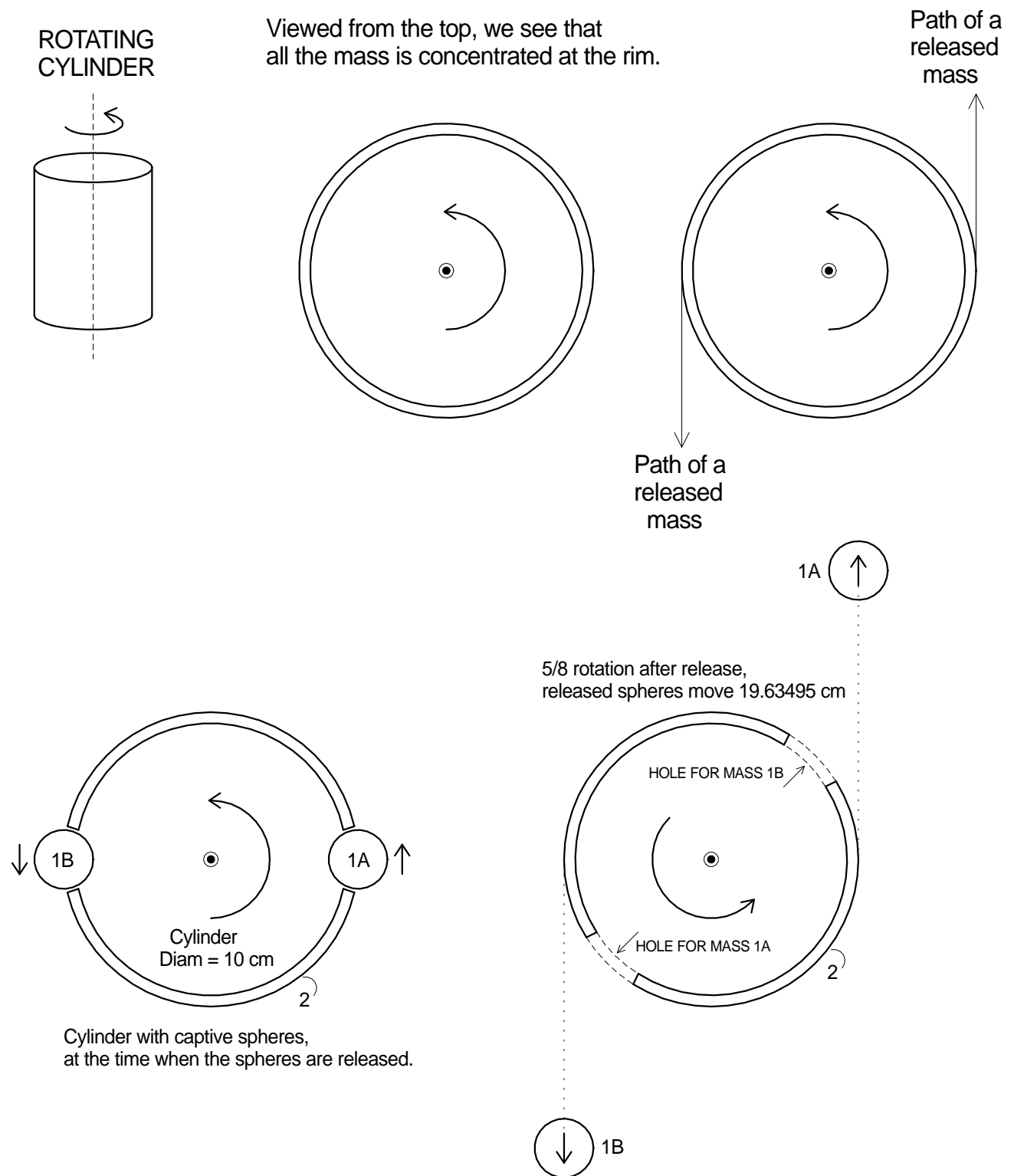
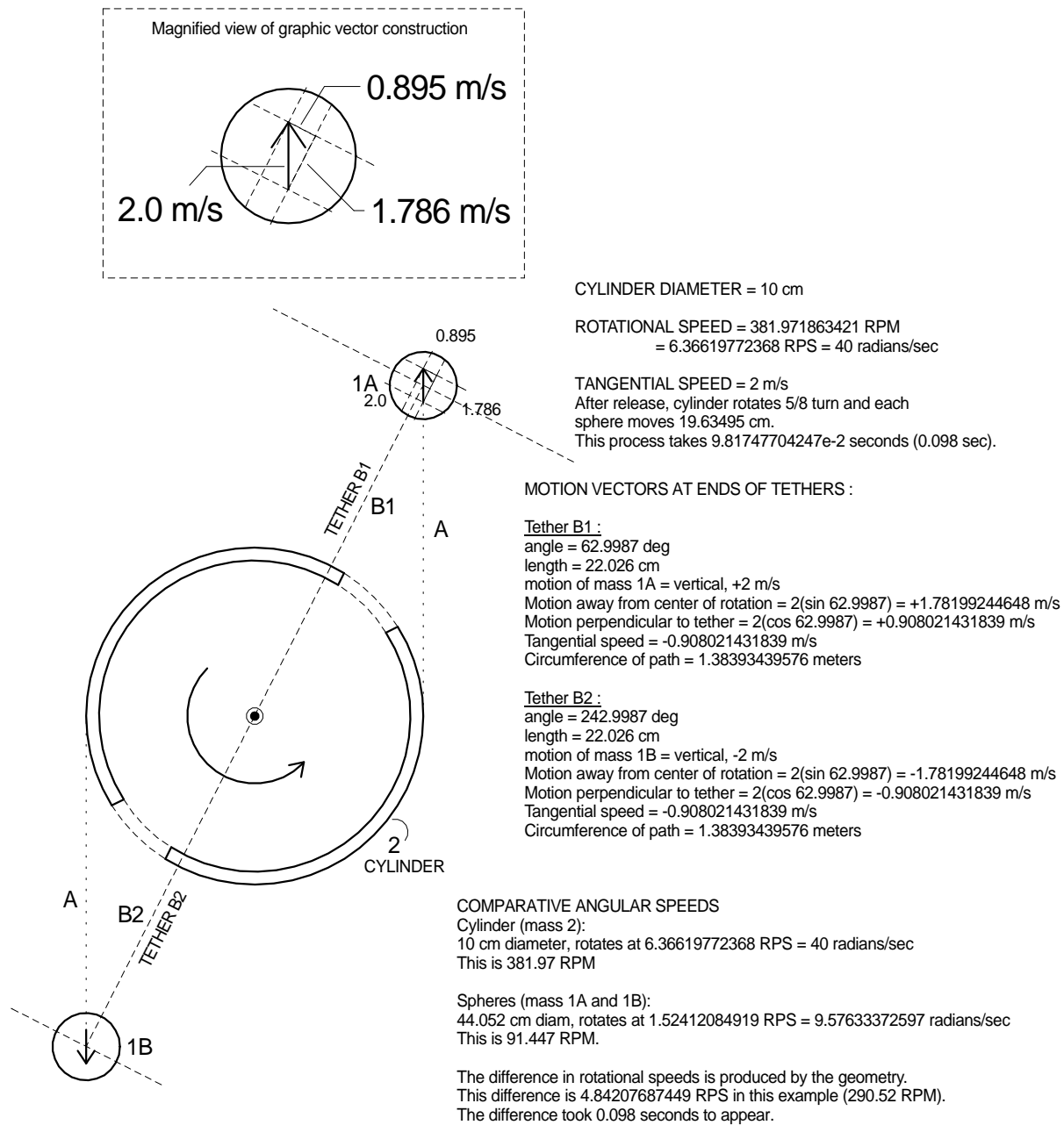


FIGURE 6



- A. 5/8 rotation after release,
released spheres move 19.63495 cm
We will estimate this graphically as 19.625 cm.
- B. Tethers tighten now, at a length of 22.026 cm each.
B1 angle to horizontal is 62.9987 degrees.
B2 angle to horizontal is 242.9987 degrees.

FIGURE 7

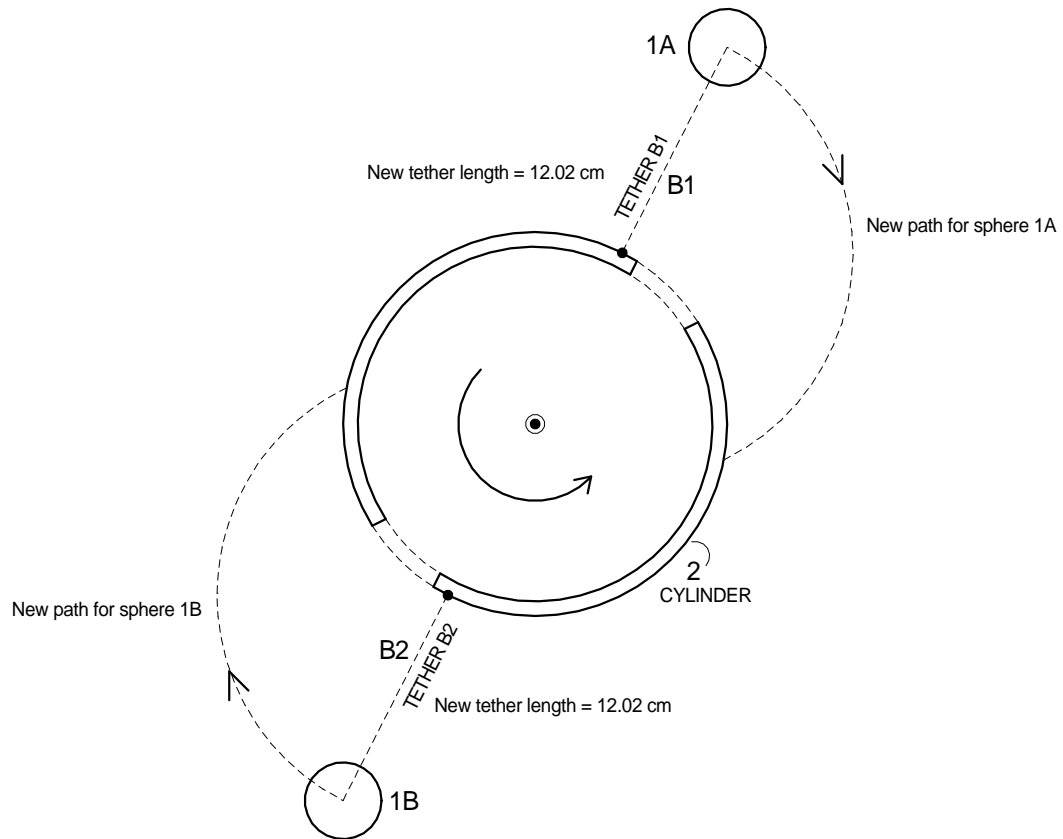


FIGURE 8